INTERNAL WAVES IN THE SEA

A summary of published information with notes on applications to naval operations

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THE PROBLEM

Investigate oceanographic factors, particularly internal waves, that influence naval operations and equipment. This report presents a bibliography of over 300 listings and summarizes important published results of investigation.

RESULTS

1. Internal waves exist in the ocean as a rule rather than an exception. Energy is transported by them in channels that are similar to sound channels. Their effect on naval exercises and equipment, particularly functions involving the use of underwater sound, can be expected to be more pronounced in these channels.

2. Although theory and observation are still not adequate for a full understanding of the phenomenon, western literature prior to June 1963 contains more than 300 publications dealing with internal waves in the atmosphere, lakes, and seas.

3. These publications are listed in the comprehensive bibliography which is part of this report.

RECOMMENDATIONS

1. Locate, compile, and keep up to date available information concerning internal waves in the sea; use both western and eastern sources.

2. Exploit this information in naval research.
ADMINISTRATIVE INFORMATION

This work was done in the Marine Environment Division under SR 004 03 01, Task 0580 (NEL L40451) from June 1963 to June 1964. The report was approved for publication 26 July 1965. The bibliography could not have been compiled without the help of the NEL library staff and D. Quist. Comments on the manuscript by G. H. Curl and E. C. LaFond are gratefully acknowledged.

CONTENTS

INTRODUCTION...page 3
  Background... 3
  Limitations on measurement... 4
EFFECTS ON UNDERWATER SOUND... 6
  Frequency channels... 6
  Breaking of internal waves... 9
  Other effects... 10
STAGES OF INTERNAL WAVES... 10
  Origin... 11
  Existence... 12
  Destruction... 15
  Direction... 18
SUMMARY... 19
BIBLIOGRAPHY... 21

ILLUSTRATIONS

1 Distribution of papers on internal waves by year... page 4
2 An example of the mode structure of internal waves... 13
3 Observations of temperature at three stations... 16, 17
INTRODUCTION

Internal waves affect the propagation of underwater sound and therefore also the U. S. Navy operational use of sonar through the phenomena of refraction and scattering. Refraction of the sound rays by internal waves can result in sonar bearing errors and in fluctuations of bearing determinations. Fluctuation in sound intensity occurs when multipath interference is present, particularly when one path passes through moving internal waves. The apparent sound attenuation may be aggravated by loss of energy from the beam by scattering from intervening internal waves. Some sound may be scattered back to the source by internal waves to increase the reverberation background of the sonar.

Background

Figure 1 shows the distribution of published papers on internal waves as a function of time. The graph shows only casual interest in internal waves after the first paper was written by G. G. Stokes in 1847. Attention increased around the year 1900, and peaks occurred circa 1910 and in the 1930's. The decline in the number of published papers after 1938 was apparently due to WWII. The last year in which no paper was published was 1944, and from then until now the number has steadily increased. Figure 1 includes papers through 1962, and the bibliography is comprehensive through June 1963.

Observational data on internal waves have been accumulating for the past 60 to 70 years. Most of these data were confined to the Scandinavian countries until about 1925. Until 1950, few of the observations on internal waves had statistical significance. Since that time, three unaliased spectra have been published to which statistical significance can be assigned. These data were observed in the Baltic Sea, off Bermuda, and off the California coast. All were taken from fixed points and consequently do not permit any study of geographical changes. Data taken from towed thermistor chains on the east and west coasts of the
Figure 1. Distribution of papers on internal waves by year.

U. S. help this problem somewhat, but interpretation of the data is difficult. For example, towing of thermistors introduces a Doppler shift into any periodic phenomenon that is in motion, and the magnitude of the shift is unknown because the directionality of internal waves is also unknown.

A large number of temperature observations in the ocean have been made for purposes other than the study of internal waves. Some of the features of these observations have been explained on the basis of internal waves, for example, the variability in dynamic heights, from which dynamic currents are computed. Changes in the relative topography of the sea surface of ±1 centimeter have been estimated for the region off Southern California.

Limitations on Measurement

The ability to measure internal waves is presently confined to the ability to measure temperature in the ocean. It is not yet possible to measure density in situ continuously. Internal waves are measured by assuming
that changes in temperature are conservative. Consequently, both salinity and density surfaces undergo the same variance over time and space.

The most widely used device for measuring temperature continuously in the ocean has been the thermistor. The thermocouple has been used with less success because it requires much more sensitive equipment for sensing and recording the output. The bathythermograph and the reversing thermometer have also been employed, but these instruments introduce aliasing. Other devices, such as platinum resistance thermometers and vibrotrons, have been utilized, but to a considerably less extent.

Current meters provide perhaps the most direct way to measure internal waves. They have had limited use, and there are a number of problems associated with the employment of them, such as motion of the device and vortex shedding, which introduce erroneous data into the records. It is desirable to overcome these problems because there are serious handicaps in measuring internal waves by the use of temperature devices.

There appears to be no lag in the development of electronic equipment capable of handling signals from these transducers and recording them. Once a method of measuring has been selected, there is no problem evident in obtaining the necessary electronic equipment to apply it, although integrating components and adapting the system for a particular use usually involve a period of several months.

Width of the spectrum and the structure of the modes must be given due regard in studies of internal waves. The latter requires that temperature measurements be made from surface to bottom. In deeper layers of the ocean, temperature gradients are weak -- as low as 0.0010° per meter. Although many claims about accuracy and stability are made, it is probable that considerable equipment development is required in order to meet the sensitivity, accuracy, and stability requirements.

In addition, it is necessary for the equipment to record for long periods of time under the rigors of the marine environment (constant motion, solvency of sea water, and extreme pressure) and with the usual power limitation of a small, isolated system.
EFFECTS ON UNDERWATER SOUND

The most direct applications of reports on internal waves are in connection with underwater sound. Numerical evaluation of the effects is known for only one case, wherein sound rays pass through a sharply defined internal wave at steep angles in shallow water. A wide knowledge of the amplitudes, wavelengths, direction of travel, and geographic and seasonal distributions of internal waves is required to evaluate the importance of these effects. Others, such as the release of gas bubbles (which scatter sound) by passage of a large-amplitude internal wave through saturated sea water, may be important.

Three fundamental factors affect the velocity of sound in sea water -- temperature, salinity, and pressure. Anything that introduces time changes or space changes in these properties will also bring in changes in the velocity of sound in the sea. The spectrum of time changes in the velocity of sound at a fixed point is directly proportional to the spectrum of time changes in any conservative property at that point. The latter spectrum can be determined from temperature measurements (provided heating at the boundaries is neglected), which are by far the easiest and most inexpensive that can be made in the ocean. In two cases an estimated 75 to 95 percent of the variance of a 3-day temperature record in shallow water was caused by internal waves. This estimate accounts for variance at frequencies of free internal waves. The range covers periods from about 5 minutes to 24 hours at 30°N.

Frequency Channels

Sound velocity in the ocean has a minimum at a depth of about 400 meters. The velocity minimum is the cause of the sound channel that has its axis at the level of the velocity minimum. Most of any omnidirectional sound released near the minimum is retained in the channel by refraction. This is a process by which direction of sound
waves constantly tends toward a velocity minimum. Sound energy concentration in the sound channel is therefore engendered by a retaining process.

Another channel for horizontal energy transmission exists in the ocean. The identity of this channel involves transverse waves and their frequency maxima. These are internal waves the amplitude of which is a function of depth and the maximum frequency of which is the Väisälä frequency (also a function of depth).

The significance of the Väisälä, or stability, frequency (usually designated by $N$) can be illustrated by considering a small particle of fluid. A bit of fluid adiabatically displaced from its zero-order position and allowed to move freely, will oscillate at the Väisälä frequency provided entropy remains constant. The bit responds to a balance between buoyant and inertial forces. Pressure inside the bit becomes that of the fluid outside. When the magnitudes of density-change inside and outside the bit are equal, it remains in its displaced position. $N$ is zero in this case and stability is neutral. When density changes inside are larger in magnitude than those outside, the bit continues its displacement without oscillating, and the fluid is unstable. The fluid is stably stratified when density change outside is greater than that inside, and the particle will oscillate about its equilibrium position after initial displacement. Application of the stability frequency to internal wave motion is immediately apparent.

Internal perturbations do not progress as free waves at frequencies above the stability frequency. This phenomenon creates energy channels or frequency channels for internal waves that are as well defined as the sound channels in the ocean.

Higher-frequency internal waves are limited to regions near the maximum of $N$, but lower frequencies are not excluded. Frequencies lower than the minimum of $N$*

*Minimum of $N$ is used for convenience with the understanding that a channel can be defined as the region between $z_1$ and $z_2$ where $N(z_1) = N(z_2)$. 

may exist at all depths, but higher frequencies are excluded from depths having low stability frequency. A wider band of frequencies existing at depths of $N_{\text{max}}$ sustains a frequency channel for internal waves by a selective process, as compared to a retaining process for the sound channel. Each frequency transports energy at its group velocity, and the frequency channel is therefore a channel for horizontal flux of energy, which is the same net effect of the sound channel.

Little is known about the relative importance of the various modes of internal waves in the ocean. On the other hand, the nature of the stability frequency is such that frequencies near the maximum of $N$ must have their largest displacements in the frequency channel. Waves of frequency lower than $N_{\text{min}}$ have no such requirement, and it is not known at what depths the various modes concentrate the horizontal flux of energy. An estimate can be based upon simple formulae. Starting with the energy equation for internal waves and the approximation

$$
\rho = \frac{\rho e^{1}}{\rho_o} \frac{d\rho_o}{dz} z,
$$

Kraus*(1964) arrived at an expression for total potential energy per unit area (horizontal) of the $n$th mode:

$$
\frac{E \text{pot}}{H} = \frac{1}{z} g \frac{1}{\rho_o} \frac{d\rho_o}{dz} a_n^n H
$$

where $a_n$ is amplitude of the $n$th mode, $H$ is total depth, $g$ is the acceleration due to gravity, $\rho_o$ is the zero-order density of seawater, and $E \text{pot}$ is total potential energy per unit horizontal area. Energy per unit volume is estimated from

$$
\frac{E' \text{pot}}{H} = \frac{1}{z} g \frac{1}{\rho_o} \frac{d\rho_o}{dz} a_n^n
$$

*Book in process
Hydrographic data (at 39° 20' N, 50° 50' W) taken from figure 7 of Fjeldstad (1933) give the following values:

\[
\frac{1}{\rho_0} \frac{d\rho}{dz} \approx 2 \times 10^{-5}
\]

below the permanent thermocline at 1300 meters and

\[
\frac{1}{\rho_0} \frac{d\rho}{dz} \approx 6 \times 10^{-4}
\]

in a rather weak permanent thermocline at 700 meters. Using these values in the equation and taking the ratio of specific energies, we have

\[
\frac{(a_7)^2}{30} \frac{n}{(a_{13})^2} \frac{n}{n}
\]

approximately, where \(a_{7n}\) and \(a_{13n}\) are the amplitudes of the \(n\)th modes at 700 and 1300 meters respectively. Thus, if displacements inside the permanent thermocline are smaller by an order of magnitude than those outside, the energies are still about the same. Evidently more energy is transmitted horizontally in the frequency channel because a wider band of frequencies is progressing there.

**Breaking of Internal Waves**

Internal waves travel horizontally, and some of them must finally contact bottom near shore. That they reflect, is a mathematical possibility, but not yet a matter of observation. Directions found by observation in very shallow water do not indicate any reflection from shore. All the energy transmitted by internal waves can end as heat, but if it is assumed for the moment that some goes into potential energy, density surfaces would be driven
upward as internal waves are destroyed near shore until other processes set in to maintain dynamic equilibrium.

Scalar surfaces tend to parallel the bottom near shore, and uplifted density surfaces near shore can be observed in cross-sectional plots of density such as those found in various atlases of oceanographic data. Horizontal gradients, in addition to vertical gradients, would be expected to be more important in coastal waters than in the deep, open sea. The importance of these horizontal gradients has not been investigated, and they are ignored in acoustic models assumed for the ocean.

Other Effects

In addition to fluctuation in sound speed and thus sound refraction, numerous other changes in the sea caused by internal waves are applicable to naval operations. These include buoyancy changes, water motions, water mixing, and displacement of water properties. Mathematical analyses have been conducted on the generation of internal waves by idealized bodies moving in stratified water. This has its obvious application to submarine detection, provided small signals can be discerned from a high background of natural internal waves.

STAGES OF INTERNAL WAVES

The studies of internal waves listed in the bibliography can be placed under three general subject headings -- origin, existence, and destruction. Most of the reports have treated one of these phases without regard for the other two.
Origin

Some theory regarding the origin of internal waves exists. One school of thought believes that internal tides originate directly from tidal forces. According to this idea, the internal waves begin at a frequency equal to the tidal frequency but at wavelengths of free waves. Another opinion contends that internal waves originate when currents pass over bottom irregularities. Origin by this process would be particularly noticeable in the region of the shelf break. Model experiments with two-layer systems indicate that internal waves start where such changes of bottom topography occur. Modeling systems without jump layers, that is, with arbitrary but stable density distribution, have evidently not been attempted.

Mathematical analysis of shear currents in the ocean leads to still another possible origin for internal waves. The analysis indicates that under certain idealized conditions, a nonbalanced velocity field in the process of adjusting toward geostrophic flow can lose energy to oscillating modes.

Although some recent progress has been made, almost nothing is known of nonlinear interactions. For example, shear currents might produce internal waves. Conversely, the question arises as to whether shear will reduce the coherence of waves as they travel through shear layers and feed energy back into the nonperiodic motion.

Some recent developments indicate that steady or low-frequency surface tides flowing over a rough bottom will originate internal waves. On the other hand, it is not known whether energy will be fed back and forth between frequencies as internal waves progress over it. From the standpoint of theory, nonlinear interaction between different kinds of motion and different frequencies is perhaps the greatest gap in present knowledge.

Both modeling and mathematical analyses indicate that internal waves can originate from atmospheric disturbances, but this possibility does not seem to be too well accepted among the investigators. All of these are reasonable theories for the origin of internal waves in
nonhomogeneous media, but adequate observations to support or discredit any theory are virtually nonexistent.

**Existence**

The simplest theory of existence of internal waves is the two-layer theory. This theory assumes a medium composed of two layers of different density. It has become clear that two-layer theory is not adequate to explain observations in the ocean except in a few special places. The special cases are largely confined to northern latitudes where fresh water overlies salt water. The Baltic Sea is a good example, but even in this instance success of the theory is limited.

A more complicated theory, which deals with existence of internal waves in a medium with continuous density distribution, is more realistic. This idea shows that it is mathematically possible for an infinite number of modes of internal waves to exist at any one frequency.

Figure 2 shows the modal distribution computed from data taken at a point off the San Diego coast. The abscissa can be regarded as amplitude in arbitrary units; \( M_1, M_2, \) etc., are the various modes. The subscripts indicate the number of the mode. The first-mode wave has a single amplitude maximum, the second mode has two amplitude maxima, and so on. It would be expected that the lower modes in the ocean would be dominant and that the higher modes would be less common because more shear is involved. This theory shows that an infinite number of modes are possible, but does not indicate the amplitudes of the modes. Amplitudes can be established only by computing from observation. Dominant modes are important and should be established, because they probably affect the coherence of waves over space.

Both two-layer theory and continuous-density theory are usually simplified by the assumptions that mean motion is constant from surface to bottom or nonexistent, and that the bottom is a plane; both assumptions are unrealistic. Turbulence, various kinds of surface waves, currents with vertical and horizontal gradients, and other motions are
known to exist. No theory accounts for the interaction between various motions and internal waves, or indeed between internal waves of different frequencies and modes. Linear superposition of different kinds of motion, including internal waves, is probably unrealistic.

Theoretical maximum and minimum frequencies for free internal waves in the ocean have been established within the last two decades. The theoretical maximum frequency is known by various names, the most common being the Väisälä frequency. The theoretical low limits are defined by the Coriolis parameter. The Väisälä frequency is expressed in terms of density, density gradient,
gravity, and sound velocity in the medium. The maximum frequency is consequently a function of depth. Its maximum occurs in the thermocline, and its lowest value at the depth where density gradient is lowest. Internal perturbations will not progress as a gravity wave at frequencies above the Väisälä frequency. Any internal oscillations at frequencies below the theoretical lowest frequency cannot exist according to the theory as free internal waves.

The phase and group velocity of internal waves is higher for lower-frequency waves. For any given frequency, first-mode internal waves have the highest phase velocity. The phase velocity of the second mode is approximately one-half the phase velocity of the first. The third-mode internal wave progresses at still lower phase velocity, and the phase velocity continues to decrease with higher mode. The mode theory of internal waves indicates that they are highly dispersive waves. Ocean currents modify phase velocity, but have little effect on the modal structure.

Only three unaliased spectra on internal temperature oscillations have been published. A continuous, monotonically decreasing spectrum seems to exist between the minimum and the maximum frequencies, but the relative proportions of variance per bandwidth contributed by internal waves and less regular motion remain unknown. This is an important point that should be emphasized for the benefit of naval operations, because wave motion is easily predictable compared to nonperiodic or transient phenomena.

The effect of rotation of the earth is to increase the amplitude of waves in a direction to the right of the direction of progress, in the northern hemisphere. This effect is apparently unimportant with reference to free internal waves. Rotation also increases the phase velocity compared to the nonrotating case.

Present-day theory on existence of internal plane waves, as opposed to origin or destruction, implies perfect coherence of plane waves over space. According to this theory, an internal wave originating on the coast of Japan travels all the way across the ocean undistorted. If this is true, changes in temperature structure caused by internal waves should be easily predictable for any point in the
ocean. Although observations of internal waves are generally lacking, studies of coherence by analysis of observed data are perhaps the most important from the standpoint of naval operations.

**Destruction**

The literature contains only one report that deals directly with the destruction of internal waves. Instability in a two-layer system depends on densities and velocities of the layers and the wave number. In shallow water where the thickness of both layers is small, instability depends also on their thickness. The criterion for instability is satisfied for values that are observed in the ocean and internal breakers can happen according to this theory.

Time-series observations of temperature at three stations in shallow water off Mission Beach, California, have been recorded for several years. In all these observations only one case has been observed that can be attributed to instability of a progressive internal wave (fig. 3). The instability in this case is more analogous to whitecaps of wind waves than to plunging breakers at the shoreline. Short-period changes in surf temperature can be detected by an attentive swimmer. They might be caused by higher-frequency internal waves such as that shown in figure 3.

Viscosity attenuates motion, and it can be accounted for with slight modification of the equations, but the effect of viscosity seems to be small in the ocean.
Figure 3A. Observations of temperature at station shown in inset.

Figure 3B. Observations of temperature at station shown in inset. Instability is evidently the unusual result of an internal breaker.

Figure 3C. Observations of temperature at station shown in inset.
Direction

Direction of progress is virtually overlooked in the literature. Directionality is important from the standpoint of application to naval operations, because it affects prediction of temperature and density structure. For example, if the progress of internal waves were known to be confined to a very narrow beam width, one would be able to predict the temperature structure, with due consideration for coherence and interactions, at points along a line parallel to the direction of the waves from observations at a point along the line.

Observations in shallow water indicate that internal waves are refracted upon contact with a shoaling bottom. The data also indicate that at least for the higher frequencies no reflection occurs at the shore. Standing internal waves in closed basins have been postulated in order to interpret temperature observations. The evidence is not conclusive in any of these cases.
SUMMARY

Internal-wave literature originating in western countries, plus a few papers from Soviet and Chinese works, totals about 350 documents. Most deal with internal sea waves, but a few papers on internal waves in the atmosphere are included to serve as a cross section of the literature. Similar theory applies to both media. The complete bibliography on internal waves in the atmosphere probably would be more extensive than the bibliography on internal waves in the ocean, because meteorology is more advanced than oceanography by several decades. A second consideration is that all pertinent U. S. S. R. and Asian literature is undoubtedly not included here. The language barrier and political conflict hamper communication to such an extent that a comprehensive bibliography on eastern literature would be impossible to compile.

Theory and observation of internal waves are incomplete and largely unrelated. Nonlinear theories of internal waves have been given little attention. Most present theory is kept linear by the neglect of higher-order terms. The theory of infinitesimal waves, very small in amplitude compared to length, is well developed. Finite-amplitude internal waves have been observed on both the west and east coasts of the United States. This directly indicates that a nonlinear theory is necessary for full understanding of internal waves in the ocean.

The equations of motion that govern internal waves are known for a medium with a continuous density distribution over depth and for mean motion that is constant over depth. Solutions of the equations have been obtained for plane boundaries or boundaries of intersecting planes. Solutions have been recently obtained for an irregular bottom by the use of perturbation methods. Internal waves originate by flow over an irregular bottom, by atmospheric disturbances, and possibly by shear flow, and theoretically exist in an infinite number of modes. Free internal waves are confined to a certain band of frequencies. The lower limit of this frequency band confines internal waves to certain limits of latitude, depending on frequency. Some
objections have been raised to the low-frequency limit, because observations of what are claimed to be free internal waves had been made at higher latitude than the theory would permit.

Increasing awareness of the importance of internal waves in naval operations is apparent among Navy scientists, but it can be developed among the masters of vessels and equipment only through proper instruction. Because present-day knowledge of internal waves is stored in print in so many widely scattered publications, a comprehensive bibliography on internal waves is considered an important bridge to more comprehensive naval research, development, and teaching for Fleet use.
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INTERNAL WAVES IN THE SEA

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This report presents a bibliography of over 300 listings and summarizes important published results of investigation. Internal waves are shown to exist as a rule, rather than as an exception. They transport energy in channels similar to sound channels. Effects on sonar are greatest in these channels.
Internal waves

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This report presents a bibliography of over 300 listings and summarizes important published results of investigation. Internal waves are shown to exist as a rule, rather than as an exception. They transport energy in channels similar to sound channels. Effects on sonar are greatest in these channels.

<table>
<thead>
<tr>
<th>SR 004 03 01, Task 0580 (NEL L40451)</th>
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CODE 1620
CODE 3710L (2)
CODE 320
CODE 360
CHIEF, BUREAU OF NAVAL WEAPONS
DLI-3
DLI-21
#AS
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BUDC=2
CHIEF, BUREAU OF YARS AND DOCKS
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PERS 110
CHIEF OF NAVAL OPERATIONS
OP-07T
OP-71
OP-635G
OP-0995
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CODE 418
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CODE 460
COMMANDER IN CHIEF US PACIFIC FLEET
COMMANDER IN CHIEF US ATLANTIC FLEET
COMMANDER OPERATIONAL TEST AND EVALUATION FORCE
DEPUTY COMMANDER OPERATIONAL TEST - EVALUATION FORCE, PACIFIC
COMMANDER CRUISER-DESTROYER FORCE, US ATLANTIC FLEET
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